

CLASS: XI: MATHEMATICS
TRIGONOMETRY FORMULAE

PRACTICE QUESTIONS ON TRANSFORMATION OF FORMULAE

FORMULA USED

Product to Sum or Difference Formulae

1. $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$
2. $2 \cos A \sin B = \sin(A + B) - \sin(A - B)$
3. $2 \cos A \cos B = \cos(A + B) + \cos(A - B)$
4. $2 \sin A \sin B = \cos(A - B) - \cos(A + B)$

Sum or Difference to Product Formulae

5. $\sin C + \sin D = 2 \sin \frac{C + D}{2} \cos \frac{C - D}{2}$
6. $\sin C - \sin D = 2 \cos \frac{C + D}{2} \sin \frac{C - D}{2}$
7. $\cos C + \cos D = 2 \cos \frac{C + D}{2} \cos \frac{C - D}{2}$
8. $\cos C - \cos D = 2 \sin \frac{C + D}{2} \sin \frac{D - C}{2}$

1. Find the value of
 - (i) $2 \sin 15^\circ \cos 75^\circ$
 - (ii) $2 \cos 45^\circ \sin 15^\circ$
 - (iii) $2 \sin 75^\circ \sin 15^\circ$
2. Find the value of $2 \cos \frac{\pi}{13} \cos \frac{9\pi}{13} + \cos \frac{3\pi}{13} + \cos \frac{5\pi}{13}$.
3. Find the value of $\sin \frac{5\pi}{12} \sin \frac{\pi}{12}$.
4. Prove that: $\sin 20^\circ \sin 40^\circ \sin 80^\circ = \frac{\sqrt{3}}{8}$.
5. Prove that: $\tan(45^\circ + \theta) - \tan(45^\circ - \theta) = 2 \tan 2\theta$
6. Prove that: $\tan 20^\circ \tan 40^\circ \tan 80^\circ = \tan 60^\circ$
7. Prove that: $\sec\left(\frac{\pi}{4} + \theta\right) \cdot \sec\left(\frac{\pi}{4} - \theta\right) = 2 \sec 2\theta$
8. Prove that $\frac{2 \sin(\alpha - \gamma) \cos \gamma - \sin(\alpha - 2\gamma)}{2 \sin(\beta - \gamma) \cos \gamma - \sin(\beta - 2\gamma)} = \frac{\sin \alpha}{\sin \beta}$.
9. Prove that $\cos(120^\circ + \alpha) \cdot \cos(120^\circ - \alpha) = \frac{2 \cos 2\alpha - 1}{4}$

10. Prove that $\tan 20^\circ \tan 40^\circ \tan 60^\circ \tan 80^\circ = 3$.

11. Prove that: $\sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{16}$.

12. Prove that $\cos^2 A + \cos^2 B - 2 \cos A \cos B \cos(A+B) = \sin^2(A+B)$

13. Prove that $\sin^2 A + \sin^2(A-B) - 2 \sin A \cos B \sin(A-B) = \sin^2 B$

14. Prove that $\tan(A+30^\circ) + \cot(A-30^\circ) = \frac{1}{\sin 2A - \sin 60^\circ}$

15. Prove that $4 \cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \frac{1}{4}$

16. Prove that $4 \cos A \cos(60^\circ - A) \cos(60^\circ + A) = \cos 3A$

17. Prove that $\sin A \sin(B-C) + \sin B \sin(C-A) + \sin C \sin(A-B) = 0$

18. Prove that $\sin(60^\circ + A) \sin(420^\circ - A) = \frac{1 + 2 \cos 2A}{4}$

19. Prove that: $(\cos \alpha + \cos \beta)^2 + (\sin \alpha + \sin \beta)^2 = 4 \cos^2 \left(\frac{\alpha - \beta}{2} \right)$

20. If $\cos A + \cos B = \frac{1}{3}$ and $\sin A + \sin B = \frac{1}{4}$, then prove that $\tan \frac{A+B}{2} = \frac{3}{4}$

21. Find the value of (i) $\sin 75^\circ + \sin 15^\circ$ (ii) $\cos 75^\circ + \sin 15^\circ$ (iii) $\cos \frac{4\pi}{5} + \cos \frac{\pi}{5}$

(iv) $\sin 50^\circ - \cos 80^\circ$

22. Find the value of $\frac{\cos 20^\circ - \cos 70^\circ}{\sin 70^\circ - \sin 20^\circ}$.

23. Find the value of $\frac{\sin 75^\circ - \sin 15^\circ}{\cos 75^\circ - \cos 15^\circ}$.

24. Show that: $\sin \left(\frac{\pi}{4} + \theta \right) + \sin \left(\frac{\pi}{4} - \theta \right) = \sqrt{2} \cos \theta$

25. Show that $\cos \left(\frac{2\pi}{3} + \theta \right) + \cos \left(\frac{2\pi}{3} - \theta \right) = \sqrt{3} \cos \theta$

26. Prove that $\cos 15^\circ - \sin 15^\circ = \frac{1}{\sqrt{2}}$

27. Prove that $\frac{\tan 5\theta + \tan 3\theta}{\tan 5\theta - \tan 3\theta} = 4 \cos 2\theta \cos 4\theta$

28. Prove that $\frac{\cos \theta + \cos 2\theta + \cos 3\theta + \cos 4\theta}{\sin \theta + \sin 2\theta + \sin 3\theta + \sin 4\theta} = \cot \frac{5\theta}{2}$

29. Prove that $\frac{\cos 3\theta + 2 \cos 5\theta + \cos 7\theta}{\cos \theta + 2 \cos 3\theta + \cos 5\theta} = \cos 2\theta - \sin 2\theta \tan 3\theta$

30. If $x \cos \theta = y \cos \left(\theta + \frac{2\pi}{3} \right) = z \cos \left(\theta + \frac{4\pi}{3} \right)$, then show that $xy + yz + zx = 0$.
31. Prove that $\cos \alpha + \cos \beta + \cos \gamma + \cos(\alpha + \beta + \gamma) = 4 \cos \frac{\alpha + \beta}{2} \cos \frac{\beta + \gamma}{2} \cos \frac{\gamma + \alpha}{2}$.
32. Prove that $\sin \alpha + \sin \beta + \sin \gamma - \sin(\alpha + \beta + \gamma) = 4 \sin \frac{\alpha + \beta}{2} \sin \frac{\beta + \gamma}{2} \sin \frac{\gamma + \alpha}{2}$.
33. Prove that $\frac{\cos 2A \cos 3A - \cos 2A \cos 7A + \cos A \cos 10A}{\sin 4A \sin 3A - \sin 2A \sin 5A + \sin 4A \sin 7A} = \cot 6A \cot 5A$
34. Prove that $\frac{\cos \left(\frac{\pi}{4} + \theta \right) - \cos \left(\frac{\pi}{4} - \theta \right)}{\sin \left(\frac{2\pi}{3} + \theta \right) - \sin \left(\frac{2\pi}{3} - \theta \right)} = \sqrt{2}$
35. Prove that $\frac{\sin(4A - 2B) + \sin(4B - 2A)}{\cos(4A - 2B) + \cos(4B - 2A)} = \tan(A + B)$
36. If $A + B + C = \pi$, prove that $\frac{\cos A}{\sin B \sin C} + \frac{\cos B}{\sin C \sin A} + \frac{\cos C}{\sin A \sin B} = 2$
37. Prove that $\sin(y + z - x) + \sin(z + x - y) + \sin(x + y - z) - \sin(x + y + z) = 4 \sin x \sin y \sin z$
38. If $b \sin \beta = a \sin(2\alpha + \beta)$, prove that $(b + a) \cot(\alpha + \beta) = (b - a) \cot \alpha$
39. If $\sin A + \sin B = a$ and $\cos A + \cos B = b$, then prove that (i) $\tan \frac{A+B}{2}$ and (ii) $\tan \frac{A-B}{2}$.
40. If $\sin \theta = n \sin(\theta + 2\alpha)$, prove that $\tan(\theta + \alpha) = \frac{1+n}{1-n} \tan \alpha$.